Spacecraft Attitude Control using NSTSM and RW Desaturation via Magnetorquer

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***Abstract*—In this paper, the problem of controlling the orientation of a rigid spacecraft in the presence of parametric uncertainty and external disturbances is examined. Non-singular terminal sliding mode (NSTSM) based control has been derived using the backstepping framework giving the system finite-time stability with continuous control inputs. NSTSM can only reject matched disturbances, therefore, the gains are made adaptive to adjust according to the upper bound of the external disturbance. Reaction wheels (RWs) are taken as primary actuators and modeled as first order system. A new RW desaturation strategy using magnetorquer has been proposed. The proposed desaturation method saves significant energy compared to conventional RW desaturation methods. Finally, the effectiveness of the control algorithm along with desaturation strategy is demonstrated by numerical simulations.**

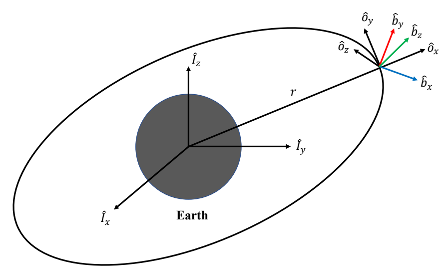

Figure 1. Frames of reference

I. Introduction

Spacecraft attitude control is a fundamental problem for any space mission. Attitude control approach that is robust to parametric and external disturbances is always desired. One such approach is sliding mode control. Sliding mode control has been used extensively for its disturbance rejection and finite-time convergence properties using fractional power state feedback [1], [2]. Conventional sliding mode control suffers from chattering phenomenon caused by switching function. But, there are ways to handle this problem such as the use of smooth switching function.
RWs are one of the most preferred type of actuators for spacecraft attitude control because of its ability to provide very precise pointing accuracy [3]. RWs suffer from a fundamental problem of saturation and can no longer exchange momentum with the spacecraft after saturation [4], [5]. Once the spacecraft reaches its desired attitude, non-zero RW angular speed keeps consuming on-board power [6]. Various desaturation methods consists of the use of cold gas thrusters, electric propulsion and magnetorquer [4]. The propulsion based methods are disadvantageous due to the requirement of gas storage on-board the spacecraft, which keeps depleting with time, on the other hand, magnetorquer only need the existence of magnetic field with sufficient strength and electric power which can be recharged using solar radiation [7]. For satellites in lower and medium earth orbit, both the requirements i.e. availability of magnetic field and solar radiation are fulfilled, therefore, this paper makes an attempt to solve the problem of attitude stabilization using RWs and desaturation of RWs using magnetorquer.

This paper presents a control algorithm for a second order strict feedback system that combines the advantages of backstepping framework such as globally asymptotic stability in the sense of Lyapunov and fractional power feedback giving finite-time convergence and robustness against external and parametric disturbances. The switching gains in the control strategy are made adaptive since the upper bound of disturbance terms is generally unknown. The control algorithm is then applied for the spacecraft attitude dynamics and a control law is given for the control torque. For desaturation of RWs, a desaturation strategy is proposed that gives the minimum required magnetic moment for a given momentum dumping rate.

II. Preliminaries

The problem of spacecraft in a circular orbit is considered in this paper. The orbital radius is 8059 km and the orbit inclination is taken as 30 deg from the equatorial plane. The attitude stabilization is done *wrt.* orbital frame which means the considered spacecraft can be pointed towards a desired direction on earth or in space. The frames of references that are used in this paper are shown in Fig. (1) where I is inertial frame with basis vector $[\hat{I}_x, \hat{I}_y, \hat{I}_z]^T$, o is the orbital frame with basis vector $[\hat{o}_x, \hat{o}_y, \hat{o}_z]^T$ and b is spacecraft body frame with basis vector $[\hat{b}_x, \hat{b}_y, \hat{b}_z]^T$.

A. Notation

In this paper, $S(x)$ is defined as follows

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

III. Kinematics and Dynamics

The system considered here is a rigid spacecraft whose attitude kinematics is written *wrt.* orbital frame and dynamics is written in inertial frame as follows

$$\dot{q_r} = \frac{1}{2} q_r \otimes \boldsymbol{\omega}_r \quad (1)$$

$$J\dot{\omega} = S(J\omega + h_w)\omega + T_{gg} + u + d \quad (2)$$

$$u = T_{rw} + T_m \quad (3)$$

$$T_{rw} = -\dot{h}_w \quad (4)$$

$$T_m = m \times B \quad (5)$$

where $q_r = [q_{1r}, q_{2r}, q_{3r}, q_{4r}]^T \in \mathbb{R}^{4\times1}$ is a unit quaternion that satisfies $q_r^T q_r = 1$, $\boldsymbol{\omega}_r \in [0, \omega_{1r}, \omega_{2r}, \omega_{3r}]^T \in \mathbb{R}^{4\times1}$ is angular velocity quaternion of the spacecraft relative to orbital frame , $\omega \in \mathbb{R}^{3\times1}$ is angular velocity vector of the spacecraft in inertial frame, $J \in \mathbb{R}^{3\times3}$ is the inertia matrix of the spacecraft, $h_w \in \mathbb{R}^{3\times1}$ is the reaction wheel angular momentum, $u \in \mathbb{R}^{3\times1}$ is control torque which is made up of two components where torque due to reaction wheel is $T_{rw} \in \mathbb{R}^{3\times1}$ and torque due to magnetorquer is $T_m \in \mathbb{R}^{3\times1}$, $m \in \mathbb{R}^{3\times1}$ is magnetic dipole moment of 3 orthogonal torque rods a, $B \in \mathbb{R}^{3\times1}$ is ambient geomagnetic field in spacecraft body frame, $T_{gg} \in \mathbb{R}^{3\times1}$ is torque due to gravity gradient and $d \in \mathbb{R}^{3\times1}$ is the external disturbance torque. Angular velocity vector ω_r *wrt.* orbital frame is given by this relation

$$\omega_r = \omega - \omega_0 \quad (6)$$
$$\omega_0 = |\omega_0| \hat{o}_z^B \quad (7)$$

where $|\omega_0| \in \mathbb{R}^1_{>0}$ is the orbital angular velocity of the spacecraft on a circular orbit and $\hat{o}_z^B$ is the z-axis of the orbital frame transformed into the body frame. The gravity gradient is modeled as following [8]:

$$T_{gg} = -3\omega_0^2 S(J\hat{o}_x^B)\hat{o}_x^B \quad (8)$$

where $\hat{o}_x^B$ is the x-axis of the orbital frame transformed into the body frame. The rotation matrix that transforms a vector from the orbital frame to the body frame is given as follows:

$$R = \begin{bmatrix} 2(q_1^2 + q_2^2) - 1 & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 - q_1q_3) \\ 2(q_2q_3 + q_1q_4) & 2(q_1^2 + q_3^2) - 1 & 2(q_3q_4 + q_1q_2) \\ 2(q_2q_4 + q_1q_3) & 2(q_3q_4 - q_1q_2) & 2(q_1^2 + q_4^2) - 1 \end{bmatrix} \quad (9)$$

RWs are modeled as first order system $\frac{1}{\tau s+1}$ with time constant τ seconds.

IV. Error kinematics

Let $q_{rd} = [q_{1_{rd}}, q_{2_{rd}}, q_{3_{rd}}, q_{4_{rd}}]^T \in \mathbb{R}^4$ be the desired quaternion attitude *wrt.* orbital frame and $q_{rd}^{-1} = [q_{1_{rd}}, -q_{2_{rd}}, -q_{3_{rd}}, -q_{4_{rd}}]^T \in \mathbb{R}^4$ is the conjugate of q_{rd}, then the quaternion error, denoted by $q_{re} \in \mathbb{R}^4$, can be written as follows [9]

$$q_{re} = q_{rd}^{-1} \otimes q_r \quad (10)$$

And, the error kinematics is derived as

$$\dot{q}_{re} = q_{rd}^{-1} \otimes (\dot{q}_r - \dot{q}_{rd} \otimes q_{re}) \quad (11)$$

Using Eq. (1) for both q_{re} and q_{rd}, Eq. (11) becomes

$$\begin{aligned} \dot{q_{re}} &= \frac{1}{2} q_{rd}^{-1} \otimes (\frac{1}{2} q_r \otimes \boldsymbol{\omega}_r - \frac{1}{2} q_d \otimes \boldsymbol{\omega}_{rd}^D \otimes q_{re}) \\ &= \frac{1}{2} q_{re} \otimes (\boldsymbol{\omega}_r - q_{re}^{-1} \otimes \boldsymbol{\omega}_{rd}^D \otimes q_{re}) \end{aligned} \quad (12)$$

Here, $\omega_{rd}^D \in \mathbb{R}^4$ is the desired angular velocity quaternion of desired reference frame. Let $\boldsymbol{\omega}_{rd}^B$ be the desired angular velocity quaternion of the body-frame, then the transformation between angular velocities in body-frame and angular velocity in the desired frame can be made through the following expression

$$\boldsymbol{\omega}_{rd}^D = q_{re} \otimes \boldsymbol{\omega}_{rd}^B \otimes q_{re}^{-1}$$

As a result, Eq. (12) becomes

$$\dot{q_{re}} = \frac{1}{2} q_{re} \otimes (\boldsymbol{\omega}_r - \boldsymbol{\omega}_{rd}^B) \quad (13)$$

By writing Eq. (13) as $\boldsymbol{\omega}_r - \boldsymbol{\omega}_{rd}^B = 2q_{re}^{-1} \otimes \dot{q}_{re}$, we get

$$\begin{aligned} \dot{\boldsymbol{\omega}}_r - \dot{\boldsymbol{\omega}}_{rd}^B &= 2\dot{q}_{re}^{-1} \otimes \dot{q}_{re} + 2q_{re}^{-1} \otimes \ddot{q}_{re} \\ &= 2(\| q_{re} \|^2, 0) + 2q_{re}^{-1} \otimes \ddot{q}_{re} \end{aligned} \quad (14)$$

The vector part of Eq. (14) can be written as

$$\dot{\omega}_r - \dot{\omega}_{rd}^B = 2G\ddot{q}_{re} \quad (15)$$

By rearranging Eq. (15), we get

$$\ddot{q}_{re} = \frac{1}{2} G^T (\dot{\omega}_r - \dot{\omega}_{rd}^B) \quad (16)$$

where

$$G = \begin{bmatrix} -q_2 & q_1 & q_4 & -q_3 \\ -q_3 & -q_4 & q_1 & q_2 \\ -q_4 & q_3 & -q_2 & q_1 \end{bmatrix}_{re}$$

It is to be noted that for a tracking problem, $\dot{\omega}_{rd}^B$ will be non-zero and it is required to compute correctly in the body-frame. Let $R(q_{re})$ be a rotation matrix that transforms a vector from body-frame to desired frame, $\omega_{rd}^B = R(q_{re})^T \omega_{rd}^D$. or

$$\dot{\omega}_{rd}^B = \dot{R}(q_{re})^T \omega_{rd}^D + R(q_{re})^T \dot{\omega}_{rd}^D \quad (17)$$

V. Control Law Design

A. Second order strict feedback system

Control law is designed using backstepping framework for a second order system written in strict feedback form because the error kinematics given in section 4 can be converted into strict feedback form. Consider the second order system given by the following equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \end{aligned} \tag{18}$$

Here, $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are the states of the second order system and $u \in \mathbb{R}$ is the control input. Our goal is to design a suitable control input u such that x_1 goes to its desired value in finite-time. Let us define an error as follows

$$e = x_1 - x_{1d} \tag{19}$$

Here, $x_{1d} \in \mathbb{R}$ is the desired value of x_1. After taking the derivative of Eq. (19) , it yields

$$\begin{aligned} \dot{e} &= \dot{x}_1 - \dot{x}_{1d} \\ &= x_2 - \dot{x}_{1d} \end{aligned} \tag{20}$$

Now, a radially unbounded quadratic Lyapunov function is defined as follows

$$V_1 = \frac{1}{2}e^2 \tag{21}$$

The derivative of the Lyapunov function yields

$$\begin{aligned} \dot{V}_1 &= e\dot{e} \\ &= e(x_2 - \dot{x}_{1d}) \end{aligned} \tag{22}$$

x_2 will act as a virtual controller for state x_1. Desired value of x_2 is taken as follows

$$x_{2d} = -k_1 e + \dot{x}_{1d} + s \tag{23}$$

where $k_1 \in \mathbb{R}_{>0}$, $s \in \mathbb{R}$ is a sliding surface which is defined in Eq. (25). Using the value of x_{2d} in place of x_2 in Eq. (22), the value of $\dot{V}_1$ becomes

$$\dot{V}_1 = -k_1 e^2 + es \tag{24}$$

The sliding surface s is defined as

$$s = \dot{e} + \beta \int_0^t \dot{e}^{\frac{p}{q}} dt \tag{25}$$

where $\beta \in \mathbb{R}_{\geq 0}$ and $1 < p/q < 2$ such that p and q are odd integers. The dynamics of sliding surface is derived by differentiating Eq. (25) which is given as follows

$$\begin{aligned} \dot{s} &= \ddot{e} + \beta \dot{e}^{\frac{p}{q}} \\ &= \dot{x}_2 - \ddot{x}_{1d} + \beta \dot{e}^{\frac{p}{q}} \\ &= u - \ddot{x}_{1d} + \beta \dot{e}^{\frac{p}{q}} \end{aligned} \tag{26}$$

Now, we take a combined Lyapunov function V_2 to design u that will ensure that $\dot{V}_2$ is negative definite

$$V_2 = V_1 + \frac{1}{2}s^2 \tag{27}$$

After differentiating the Lyapunov function, it yields

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + s\dot{s} \\ &= -k_1 e^2 + es + s(u - \ddot{x}_{1d} + \beta \dot{e}^{\frac{p}{q}}) \end{aligned} \tag{28}$$

Let's consider the control law u given as

$$u = -e + \ddot{x}_{1d} - \beta \dot{e}^{\frac{p}{q}} - \hat{k}_2 sign(s) \tag{29}$$

where $\hat{k}_2 \in \mathbb{R}_{>0}$ and it is an adaptive gain whose adaptation rule will be derived later. By substituting the control law u given in Eq. (29) into Eq. (28), we get

$$\dot{V}_2 = -k_1 e^2 - \hat{k}_2 s.sign(s) \tag{30}$$

Now, we derive the adaptation rule for $\hat{k}_2$. The error between the true value k_2 and its estimated value $\hat{k}_2$ is defined as follows

$$\tilde{k}_2 = k_2 - \hat{k}_2 \tag{31}$$

By differentiating Eq. (31), we get

$$\dot{\tilde{k}}_2 = -\dot{\hat{k}}_2$$

Since the true value $k_2 \in \mathbb{R}_{>0}$ is unknown but it is assumed constant therefore its derivative is zero. Let us define the third Lyapunov function so that we can determine a stable adaptation rule for $\hat{k}_2$.

$$V_3 = V_2 + \frac{1}{2\gamma}\tilde{k}_2^2 \tag{32}$$

By taking the derivative of the above Eq. (32), we get

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \frac{1}{\gamma}\tilde{k}_2 \dot{\tilde{k}}_2 \\ &= -k_1 e^2 - (k_2 - \tilde{k}_2)s.sign(s) - \frac{1}{\gamma}\tilde{k}_2 \dot{\hat{k}}_2 \\ &= -k_1 e^2 - k_2 s.sign(s) + \tilde{k}_2(|s| - \frac{\dot{\hat{k}}_2}{\gamma}) \end{aligned} \tag{33}$$

The following adaptation rule is taken for $\hat{k}_2$

$$\dot{\hat{k}}_2 = \gamma |s| \tag{34}$$

From the Eqs. (33) and (34), we get

$$\dot{V}_3 = -k_1 e^2 - k_2 |s| \leq 0$$

The overall Lyapunov function V_3 is negative definite, therefore the system is stable for the designed control law u and the adaptation rule $\dot{\hat{k}}_2$.

B. Spacecraft attitude control

Rewriting the error kinematics given in Eqs. (13) and (16) below

$$\begin{aligned}\dot{q}_{re} &= \frac{1}{2}q_{re} \otimes (\boldsymbol{\omega}_r - \boldsymbol{\omega}^B_{rd}) = q_{rw} \\ \dot{q}_{rw} &= \ddot{q}_{re} = \frac{1}{2}G^T(\dot{\omega}_r - \dot{\omega}^B_{rd}) = u_1 \end{aligned} \tag{35}$$

Equation (35) is a second order strict feedback system expressed more clearly below

$$\begin{aligned}\dot{q}_{re} &= q_{rw} \\ \dot{q}_{rw} &= u_1\end{aligned} \tag{36}$$

Our goal here is to design the virtual control u_1 such that attitude quaternion and angular velocity approaches to the desired attitude quaternion and desired angular velocity. Actual control torque u is related to the virtual control u_1 with the following relation

$$u_1 = \frac{1}{2}G^T\left(\left(J^{-1}(S(J\omega + h_w)\omega + T_{gg} + u\right) - \dot{\omega}^B_{rd}\right) \tag{37}$$

Since, we have already designed a control law for a second order system, we need not repeat the derivation here. We refer to Eq. (29) for writing the u_1

$$u_1 = -q_{re} + \ddot{q}_{rd} - \beta\dot{q}_{re}^{\frac{p}{q}} - \hat{k}_2 sign(s) \tag{38}$$

Here, $\beta \in \mathbb{R}^{4\times4}$, $\hat{k}_2 \in \mathbb{R}^{4\times4}$ and $\dot{q}_{re}^{\frac{p}{q}} = \begin{bmatrix} \dot{q}_{1_{re}}^{\frac{p}{q}} & \dot{q}_{2_{re}}^{\frac{p}{q}} & \dot{q}_{3_{re}}^{\frac{p}{q}} & \dot{q}_{4_{re}}^{\frac{p}{q}} \end{bmatrix}^T$. The terms have the same meanings as described previously. Sliding surface $s \in \mathbb{R}^{4\times1}$ for this system is written as follows

$$s = \dot{q}_{re} + \beta\int_0^t \dot{q}_{re}^{\frac{p}{q}} dt \tag{39}$$

The adaptation rule for $\hat{k}_2$ is given similar as earlier

$$\dot{\hat{k}}_2 = \gamma|s| \tag{40}$$

Here, $\gamma \in \mathbb{R}^{4\times4}_{>0}$ is a diagonal matrix and $|s| = \begin{bmatrix} |s_1| & |s_2| & |s_3| & |s_4| \end{bmatrix}^T$. By using the Eqs. (37) and (38), we can write the control torque u as follows

$$\begin{aligned} u = 2G&\left(-q_{re} + \ddot{q}_{rd} - \beta\dot{q}_{re}^{\frac{p}{q}} - \hat{k}_2 sign(s)\right) + ... \\ &+ \dot{\omega}^B_{rd} - S(J\omega + h_w)\omega - T_{gg}\end{aligned} \tag{41}$$

It should be noted that during implementation of this control law, $sign(.)$ function is replaced with $tanh(.)$ function to make the control law continuous with a disadvantage of larger convergence time.

VI. Reaction Wheel Desaturation Strategy

Typical desaturation strategy involves slowing RWs down to acceptable level or even to zero angular speed. The RW deceleration will generate a torque on spacecraft which will change the attitude. Since, attitude is required to be stabilized during RW desaturation process, therefore, magnetorquer rods are used for generation of external torque to cancel the RW torque during desaturation process.

In this paper, we propose an energy efficient desaturation technique. The technique is very straightforward and intuitive. Since, the magnetorquer rods can provide torque only in the plane perpendicular to the local geomagnetic field shown in the Fig. (2) below. Net angular momentum h of the RWs at any given time instant can be decomposed into two orthogonal parts, $h_{\|}$ and $h_{\perp}$, where $h_{\|}$ is the component of angular momentum in the direction of magnetic field and $h_{\perp}$ lies in the plane perpendicular to magnetic field. From linear algebra the decomposition can be done as following

$$h_{\|} = \hat{b}\hat{b}^T h \tag{42}$$

$$h_{\perp} = S(\hat{b})^T S(\hat{b})h \tag{43}$$

where $\hat{b} = \frac{B}{\|B\|}$ is the unit vector of the magnetic field. At any given time instant, the torque generated by magnetorquer can desaturate only the $h_{\perp}$ component of the angular momentum. The desaturation rate is given using proportional control as follows

$$\dot{h} = -kh_{\perp} \tag{44}$$

where $k > 0$ determines the rate of momentum dumping. Since, the $\dot{h}$ will generate a torque on the spacecraft, an external torque of equal magnitude and opposite direction needs to be applied to maintain the attitude. This torque will come from the magnetorquer.

$$T_m = -\dot{h} \tag{45}$$

The minimum value of magnetic moment magnitude $|m|$ can given using $T_m = m \times B = |m||B|sin(\theta)$, where θ is the angle between m and B.

$$|m||B|sin(\theta) = k|h_{\perp}| \tag{46}$$

$$|m| = \frac{k|h_{\perp}|}{|B|sin(\theta)} \tag{47}$$

For $\theta = \frac{\pi}{2}$ rad, minimum value of magnetic moment is given by

$$|m| = \frac{|kS(\hat{b})^T S(\hat{b})h|}{|B|} \tag{48}$$

Direction of magnetic moment m is given by $\hat{h}_{\perp} \times \hat{b}$.

The dipole model of geomagnetic field $B = [B_x, B_y, B_z]^T$ is used which is given in orbital frame as follows:

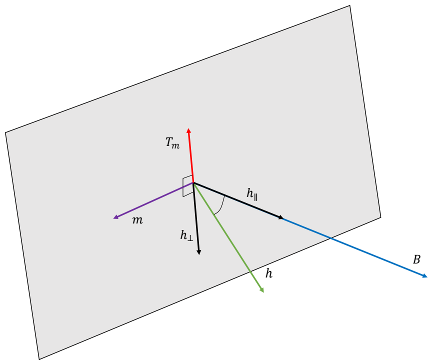

Figure 2. Decomposition of RW angular momentum h

Quantity	Value(t=0)
Quaternion	$[0.7779, 0.3185, 0.3107, -0.4437]^T$
Angular velocity (rad/s)	$[0.1, 0.2, -0.3]^T$
$\hat{k}_2$	$diag\{1, 1, 1, 1\}$

Table I
INITIAL CONDITIONS

$$B_x = B_0 cos(\omega_0 t) sin(i), B_y = B_0 cos(i), \qquad (49)$$
$$B_z = -2B_0 sin(\omega_0 t) sin(i)$$

where $B_0 = 10^{-5}$, $i = \frac{\pi}{6}$ rad is the orbit inclination angle and $\omega_0 = 8.7266 \times 10^{-4}$ rad/s orbital angular speed of the spacecraft.

VII. SIMULATION

The simulations are done in two parts. The first part simulates the attitude control and the second part simulates the desaturation of RWs using magnetorquer. The magnetorquer can be activated based on some static-rule or condition imposed on RW rpm. The simulation initial conditions and parameters are given in Table 1 and Table 2.

A. Attitude stabilization using RW

The attitude of the spacecraft is stabilized using RWs as primary actuators. The goal of attitude stabilization is to align the spacecraft to the orbital frame and keep at that attitude. The desired quaternion *wrt.* orbital frame is set to

Parameters	Value
k	10^{-4}
β	10^{-3}
p	5
q	3
$J(Kg.m^2)$	$\begin{pmatrix} 10 & -1 & -2 \\ -1 & 30 & -3 \\ -2 & -3 & 20 \end{pmatrix}$
γ	$diag\{1, 1, 1, 1\}$
$\omega_0(rad/s)$	8.7266×10^{-4}
$\tau(s)$	0.3

Table II
SIMULATION PARAMETERS

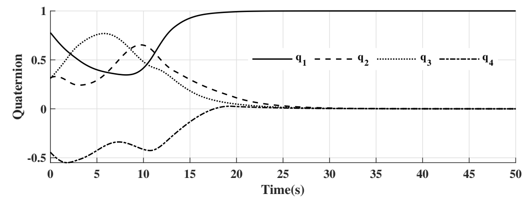

Figure 3. Attitude stabilization

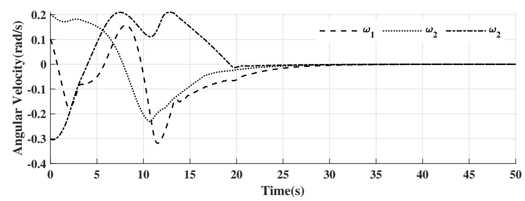

Figure 4. Angular velocity stabilization

$q_d = [1, 0, 0, 0]^T$. The results are shown from Fig. (3) to Fig. (8) for attitude stabilization. Fig. (3) shows that the desired quaternion is reached in almost 30 seconds. This is a large angle maneuver as the initial attitude quaternion if expressed in Euler angles corresponds to -50 deg yaw, 50 deg pitch and 20 deg roll. Initial angular velocity is also high which is subsequently driven to zero by control torques provided by RWs as shown in Fig. (4). The angular velocity plotted in Fig. (4) is expressed *wrt.* orbital frame. The control torques for this maneuver are shown in Fig. (5). It can be clearly seen that the torques are saturated at 1 N-m.
It should be noted that the desaturation of RWs is active, while attitude stabilization is being executed. There is a possibility that the desaturation and stabilization goals are acting counter to one another. This was a deliberate choice to conduct the simulation while desaturation algorithm is also active to demonstrate the robustness of the control algorithm. A much better approach to save energy would have been to either activate the desaturation procedure after stabilization goal is achieved or one can think of a dynamic algorithm that decides either to desaturate the RWs or act towards stabilization along with RWs based on some criteria such as RW rpm or while disturbance is acting on the spacecraft. The magnitude of magnetic moment and the torque due to magnetorquer that are acting on the spacecraft while stabilization is being performed are shown in Fig. (7) and (8) respectively. The magnitude of gravity gradient torque acting on the spacecraft is on the order of 10^{-5} N-m and it again demonstrates the robustness of control algorithm against external disturbances.

B. RW desaturation using magnetorquer

A decay rule for RW angular momentum component in a plane perpendicular to the magnetic field is proposed in this paper. Fig. (9) shows the decaying RW rpm. The simulation was conducted for 70000 seconds which is equal to 9.72 times of the orbital period. Fig. (10) and (11) shows the

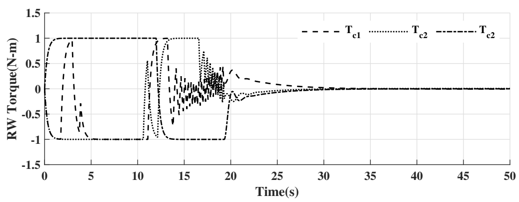

Figure 5. RW torque during attitude stabilization

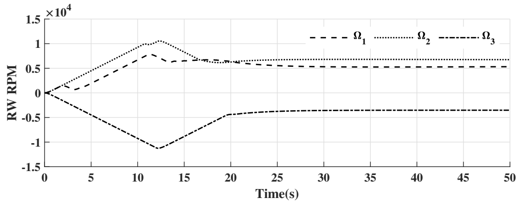

Figure 6. RW RPM

magnetic moment and torque due to magnetorquer. The decay rate in this simulation is chosen such that it gives practically realizable values for magnetic moment requirement. The angular momentum decay rate can be optimized for minimum total energy consumption during the desaturation process.

VIII. Conclusion

This paper presented the attitude stabilization of a spacecraft using RWs as actuators using NSTM based control algorithm derived using backstepping framework. The attitude stabilization performance of the control algorithm is found to be robust against external bounded disturbances. RWs while performing attitude control maneuver get saturated which are then desaturated using magnetorquer rods. The desaturation algorithm proposed in this paper computes the minimum magnetic moment that can dump the maximum angular momentum of the RW at any given point in time. However,

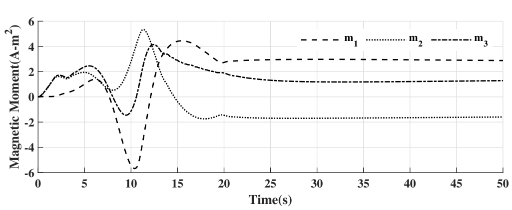

Figure 7. Magnetic moment

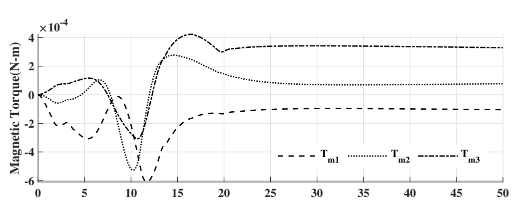

Figure 8. Torque due to magnetorquer

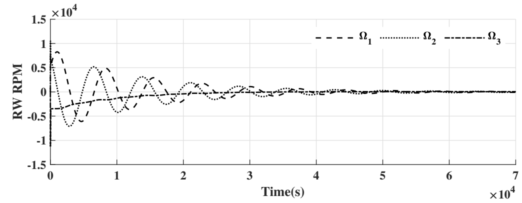

Figure 9. RW RPM

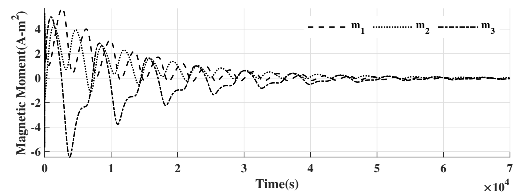

Figure 10. Magnetic moment

as the future work the rate at which angular momentum is dumped can be further optimized for a circular and elliptical orbit by minimizing the total energy consumption by RWs and magnetorquer rods.

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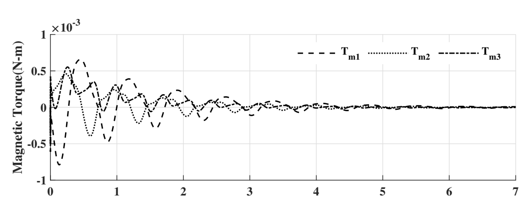

Figure 11. Torque due to magnetorquer